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An Improved Technique for the Measurement of Dynamic Mechanical Properties of Viscoelastic Materials

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Synopsis

An improved technique for the precision measurement of dynamic mechanical properties of viscoelastic materials is described. The instrumentation has been adapted for use with the commercial device Rheovibron, but can be used with any other similar device. An analysis of the technique, together with typical results, are presented. Analyses of error are included in the appendices.

INTRODUCTION

Many polymers have very low dynamic losses in temperature regions far below the glass transition temperature. In a recent study on several mixtures of poly(2,6-dimethyl-*p*-phenylene oxide) and polystyrene, the loss tangent tan δ was found to be less than 10^{-2} at low temperatures.[1] Using the commercially available dynamic-mechanical testing instrument Rheovibron (Model DDV-II-B, Toyo Baldwin Co., Japan), the resolution and the scatter of the results are such that interpretation is impossible. In this paper, we describe a new method to measure the dynamic mechanical properties of solid polymers. Although the modifications are on the Rheovibron, this method is generally applicable to the precision measurement of small phase shifts between two sinusoidal signals.

In the method to be described, we have retained the test bench of the Rheovibron, which includes the electromagnetic driver and the stress and strain gauges. We have modified the electronics by (a) providing closed-loop control so that the oscillation amplitude is constant, and (b) connecting the stress and strain gauges to a pair of low-noise chopper stabilized transducer conditioners so that stable, clean signals can be monitored continuously. We have also changed the analysis of the stress and strain signals from the "direct reading" manner of the Rheovibron with a more sensitive and simpler technique for the determination of the loss tangent and the storage modulus.

In the following section, we detail the experimental setup. In the subsequent sections, we present an analysis of our technique, and some results to illustrate the enhanced sensitivity obtained by this new technique. In Appendices I and II, we compute the errors involved in the measurements and show that they are indeed negligibly small. Finally, in Appendix III, we compare the modulus

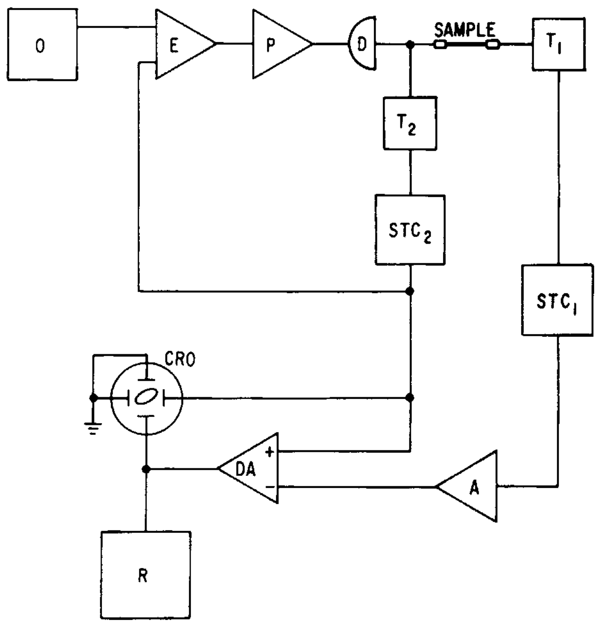

Fig. 1. Block diagram of the instrumentation: O = oscillator (0.01 Hz–1 KHz); E = error amplifier; P = dc power amplifier; D = driver; T_1 = stress gauge; T_2 = strain gauge; STC_1 = stress transducer conditioner; STC_2 = strain transducer conditioner; A = variable gain amplifier; DA = differential amplifier; CRO = oscilloscope; R = recording or measuring device.

measurements of the original Rheovibron and our modified version and show how we obtain the storage modulus E' directly.

EXPERIMENTAL SETUP

All the electronics with the original Rheovibron were replaced. The only parts that remain are (1) the bench, (2) the electromagnetic driver, (3) the stress and strain gauges, and (4) the sample positioning mechanism.

The electronics block diagram is shown in Figure 1. The instruments and the function they perform are described below:

1. Oscillator: Kikusui Electronics Corp. (Japan) Model 455. This oscillator generates a low-distortion, constant amplitude (better than 1 part in 4000) sine wave over the frequency range of 0.01 Hz to >1 KHz. It is used to provide a reference signal for the error amplifier.

2. Error Amplifier: MTS Corp. Model 440.13. This amplifier compares the reference signal from the oscillator with the strain signal from the strain transducer conditioner. The output of this amplifier is proportional to the difference signal such that the difference is driven toward zero. It enables us to maintain a constant oscillation amplitude irrespective of specimen stiffness as long as the amplitude is within the range of excursion allowed by the electromagnetic driver. Because of this strain feedback control, a constant gain on the strain signal is used. Once the strain amplitude is set, no further gain adjustments are necessary during subsequent phase measurements.

3. Power Amplifier: Hewlett Packard Model 6824A. This amplifier can provide ±1.0 amp at ±60 V from dc to >1 KHz. It converts the voltage signal

from the error amplifier into a proportionate current which excited the electromagnetic driver.

4. Transducer Conditioners: MTS Corp. Model 440.21. These conditioners provide dc excitation as well as amplification for the strain gauges. The dc excitation is constant to better than 0.01%. Amplification is chopper stabilized. The amplifiers have been modified to limit frequency response to −3 dB at 1 KHz in order to reduce the broad band noise. The two amplifiers are adjusted so that their relative phase shift is below the detection level.

5. Variable-gain Amplifier: Burr–Brown Model 3088. This is an instrumentation amplifier with relatively low noise and low drift. It is connected to the output of the stress transducer amplifier in order to vary the amplitude of the stress signal. This enables the establishment of the "horizontality" condition, to be defined in the next section.

6. Differential Amplifier: Burr–Brown Model 3088. This instrumentation amplifier amplifies the difference between stress and strain signals.

7. Amplitude Measurement: Nicolet Model 1090. This is a digital oscilloscope which can store two signals simultaneously in digital form. It incorporates a fast analog-to-digital converter (12 bit word/μsec) and a 4096-word memory. The signal amplitudes are easily determined digitally except where the signal-to-noise ratio is ~2. It is used to measure the amplitude of the difference signal.

The resolution and accuracy of our instrumentation is limited by two factors: 60 Hz noise and broad-band noise. The former can be minimized by careful grounding and shielding. The latter can be reduced by inserting a passive low-pass filter between the differential amplifier and the measuring devices. The error introduced by using a filter is small and will be discussed in Appendix II. In its present form, the instrument has a single point probable error of $\tan\delta = \pm 5 \times 10^{-4}$. The above-mentioned noise, however, limits the resolution to a minimum resolvable loss tangent value of 1.0×10^{-3}. For $\tan\delta > 2 \times 10^{-3}$, this noise is not a problem.

In the results section, we show the results of $\tan\delta$ measurement to demonstrate the resolution of the instrument.

METHOD

In this section, we describe the modified phase-shift determination that we have implemented for the Rheovibron. We have, for strain cycling at angular frequency ω,

$$\epsilon(t) = A\cos\omega t \tag{1}$$

and

$$\sigma(t) = B\cos(\omega t + \delta) \tag{2}$$

where δ is the desired phase shift between the stress and strain signals.

The difference of the stress and strain signals is given by

$$\begin{aligned}\Delta(t) &= C\cos(\omega t + \beta) \equiv \epsilon(t) - \sigma(t)\\ &= (A - B\cos\delta)\cos\omega t + B\sin\delta\sin\omega t \end{aligned}\tag{3}$$

The Rheovibron uses this difference signal to obtain tan δ. This is accomplished by adjusting the gain on the stress signal until the amplitudes of the stress and strain signals are equal,[2]

$$A = B \tag{4}$$

a condition which is determined by taking appropriate RMS readings. When eq. (4) holds, the amplitude of the difference signal, $C_{A=B}$, is given by

$$\frac{C_{A=B}}{A} = 2 \sin \frac{\delta}{2} \tag{5}$$

where we have divided by A the amplitude of the strain signal.

The Rheovibron has a calibrated scale that converts the amplitude reading obtained in eq. (5) directly to tan δ; hence it is called a "direct reading" Rheovibron.

If we, on the other hand, adjust the gain on the stress signal until we obtain the condition

$$A = B \cos \delta \tag{6}$$

as opposed to that given by eq. (4), we obtain

$$\frac{C_{A=B\cos\delta}}{A} = \frac{B \sin \delta}{A} \tag{7}$$

where we have again divided by A the amplitude of the strain signal.

When we substitute eq. (6) for A, we obtain the very elegant result

$$\frac{C_{A=B\cos\delta}}{A} = \tan \delta \tag{8}$$

Equation (8), which should be compared with eq. (5), thus enables an exact measurement of tan δ.

In order to implement this exact measurement, the condition specified in eq (6) must be satisfied. We note that when this condition holds, the strain signal $\epsilon(t)$ and the difference signal $\Delta(t)$ are 90° out of phase. This condition is thus easily detectable by inputting these signals across the x- and y-axes of an oscilloscope, respectively. The semimajor axis of length A will then lie horizontal (parallel to the x-axis) when the gain of the stress signal is adjusted until eq. (6) holds. We, therefore, call this the "horizontality" condition. The semiminor axis is then given by $B \sin \delta$. When this is divided by A, we obtain an exact expression for the loss tangent, as given in eq. (8). Although one could theoretically measure the semiminor and semimajor axes of the ellipse on the oscilloscope, we use instead a Nicolet Model 1090 digital storage oscilloscope to record the difference signal and measure the amplitude, $C_{A=B\cos\delta}$. The amplitude of the strain signal, A, is held constant using a servo-feedback loop as discussed earlier and hence need be measured only once. In Appendix I, we show that a small error in establishing "horizontality" leads to a negligibly small error in the measured tan δ. Furthermore, in Appendix II, we show that an additional phase shift introduced by passing the $\Delta(t)$ signal through a low-pass filter does not lead to a detectable error in the establishment of the "horizontality" condition.

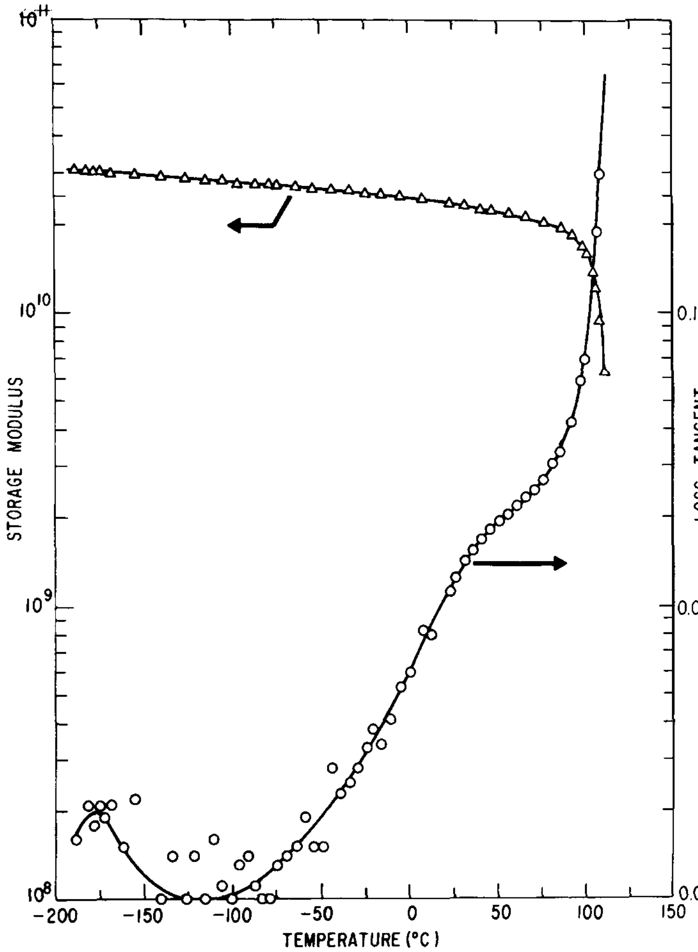

Fig. 2. Typical example of data taken on a low loss material, a 10/90 mixture of poly(2,6-dimethyl-*p*-phenylene oxide) and polystyrene.

RESULTS

We present in Figure 2 the results of one experiment which involves extremely small tan δ. The material is a 10/90 mixture of poly(2,6-dimethyl-*p*-phenylene oxide) and polystyrene. Details of the experiment and specimen preparation have been presented elsewhere.[1] The resolution limit caused by the noise is seen to be 1×10^{-3}. This is clearly shown in the temperature interval between −60° and −160°C. The existence of a peak at −180°C is suggested because the −190°C point represents several repeated measurements. The nature of this peak is discussed elsewhere.[1]

It should be noted that the level of noise is also somewhat dependent on the geometry and stiffness of the specimens. At certain temperatures, the specimen resonates with ambient vibrations emitted by humans as well as machines. It may become necessary to increase the tension on the specimen on such occasions. If the tan δ is changing rapidly, as is the case presented here, this type of noise is not usually a problem.

It should also be noted that all specimens were subjected to a pretension of about 0.2% to 0.3% steady strain.

CONCLUSIONS

As stated in the introduction, this paper arose out of our efforts to upgrade the Rheovibron. We feel that the "horizontality" method which we have presented has given us a first step in this direction and has provided many other benefits as well. Here we present the following achievements:

1. **Accuracy.** The "horizontality" condition provides a more sensitive determination of the loss tangent. A single adjustment is needed to establish "horizontality," and this measurement is attainable to a high degree of accuracy (See Appendix I). In the original Rheovibron approach, two RMS readings are required, to set $A = B$ in eq. (4), using a procedure more susceptible to larger errors. The results of the previous section bear out the increased sensitivity.

2. **Simplicity.** A measurement of "horizontality" requires a single adjustment of the gain of the stress signal amplifier, as opposed to the multiple manipulations required for the Rheovibron,[2] thus providing for ease of operation.

3. **No Unusual Equipment Necessary.** Our modification basically requires an x–y oscilloscope and some means of measuring the amplitude of the difference signal, for example, an rms meter, an oscilloscope, a strip chart recorder, a data logger, etc. This equipment is commonly available in most academic and industrial laboratories.

4. **Monitoring Capability.** The ellipse on the x–y oscilloscope is an excellent tool for monitoring the state of the sample. Insufficient tension, sample warpage, misalignment, or excessive noise are readily observable. In the original Rheovibron procedure, an oscilloscope is used in two ways: (a) to display the stress and strain signals to check for proper tension and *gross* warpage or misalignment; (b) to adjust the phase between oscillator and stress or strain amplitude measuring circuit. In our procedure, the oscilloscope displays the tan δ signal itself and is therefore much more sensitive to small warpages and misalignment.

5. **Amenable to Digital Processing.** The stress and strain signals may be directly digitized and analyzed to obtain the dynamic mechanical properties. In a paper to be published shortly, we will describe our digital analysis. This is the first step toward automation of the Rheovibron using microprocessor or computer technology.

6. **Understanding.** Finally, we feel that an important by-product of our approach is that it provides a better understanding of the procedure and the analysis of the dynamic mechanical measurements which might not otherwise be so readily apparent to the user of the Rheovibron.

The advantages of our technique are, to reiterate, fewer adjustments and switching operations, continuous signal monitoring to prevent "black box" induced errors, and higher resolution.

Appendix I

In this Appendix, we examine the uncertainty introduced in evaluating the loss tangent, tan δ, of eq. (8) due to an uncertainty in establishing the "horizontality" condition shown in eq. (6).

If the $x(t)$ and $y(t)$ signals on the oscilloscope (the strain and difference signals of Eqs. (1) and (3) are given by

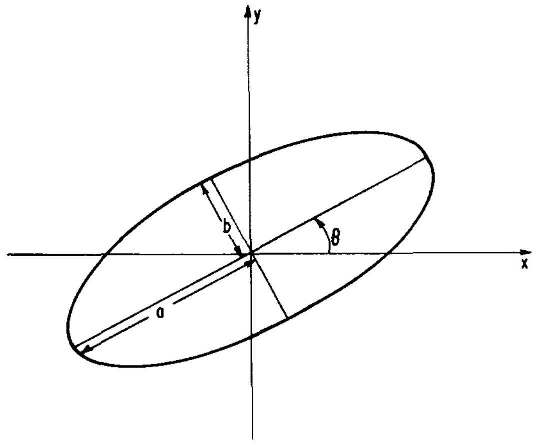

Fig. 3. General form of an ellipse: a = semimajor axis; b = semiminor axis; θ = angle with respect to x axis.

$$x(t) = A \cos wt$$

$$y(t) = A' \cos wt + B' \sin wt \tag{I-1}$$

where

$$A' = A - B \cos \delta$$

$$B' = B \sin \delta$$

then the resultant ellipse is obtained by eliminating t:

$$C_1 x^2 + C_2 2xy + C_3 y^2 = 1 \tag{I-2}$$

where

$$C_1 = \frac{1}{A^2}\left[1 + \left(\frac{A'}{B'}\right)^2\right]$$

$$C_2 = \frac{-A'}{AB'^2}$$

$$C_3 = \frac{1}{B'^2}.$$

The general form for an ellipse with semimajor and semiminor axes of lengths, a and b and which is tilted at angle θ with respect to the x-axis (see Fig. 3) is given by

$$D_1 x^2 + D_2 2xy + D_3 y^2 = 1 \tag{I-3}$$

where

$$D_1 = \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}$$

$$D_2 = \left(\frac{1}{a^2} - \frac{1}{b^2}\right) \sin \theta \cos \theta$$

$$D_3 = \frac{\sin^2 \theta}{a^2} + \frac{\cos^2 \theta}{b^2}.$$

We can now compare the special ellipse of interest given by eq. (I-2) with the general ellipse given by eq. (I-3) by matching coefficients ($D_i = C_i$ for $i = 1, 2, 3$). We solve for a, b, and θ in terms of A, A', and B' and obtain

$$\tan 2\theta = \frac{2AA'}{A^2 - A'^2 - B'^2} \tag{I-4}$$

and

$$\left.\begin{matrix}\dfrac{1}{a^2}\\ \dfrac{1}{b^2}\end{matrix}\right\} = \frac{1}{2}\left[\frac{1}{B'^2} + \frac{A'^2 + B'^2}{A^2B'^2}\right] \mp \frac{A'}{AB'^2}\left[1 + \left(\frac{A^2 - A'^2 - B'^2}{2AA'}\right)^2\right]^{1/2}$$

Let us examine two cases:

(i) In the first case, we assume that we have adjusted the amplitudes of the stress and strain signals (B and A) so that

$$\theta = 0 \tag{I-5}$$

that is, we attain perfect "horizontality" of the ellipse. Thus, from eq. (I-4),

$$2AA' = 0$$

Since $A \neq 0$,

$$A' = 0$$

or, from eq. (I-1),

$$A = B \cos \delta \tag{I-6}$$

which is, of course, our "horizontality" condition. Also, $a = A$ and $b = B'$ as expected.

(ii) Now, let us examine the more interesting case where θ is very small, but nonzero. This represents a deviation from "horizontality" by a small amount. From eqs. (I-1) and (I-4) we obtain

$$\tan 2\theta = \frac{2A(A - B\cos\delta)}{(2AB\cos\delta) - B^2} \tag{I-7}$$

Now let us define η by

$$\eta \equiv \frac{A - B\cos\delta}{B} \ll 1 \tag{I-8}$$

where η is very small since we are very close to the "horizontality" condition, eq. (I-6). Then,

$$\tan 2\theta = \frac{2(\eta + \cos\delta)\eta}{2(\eta + \cos\delta)\cos\delta - 1}$$

and to lowest order in η,

$$\tan 2\theta \simeq \frac{2\cos\delta}{\cos 2\delta}\,\eta$$

Therefore,

$$\eta \simeq \frac{\cos 2\delta}{2\cos\delta}\tan 2\theta \tag{I-9}$$

Now, in our measurement for the loss tangent, we take the ratio

$$\frac{C_{A \simeq B\cos\delta}}{A} = \frac{(A'^2 + B'^2)^{1/2}}{A}$$

which, using eqs. (I-1) and (I-8), becomes

$$\frac{C_{A \simeq B\cos\delta}}{A} = \frac{\tan\delta\left[\left(\dfrac{\eta}{\sin\delta}\right)^2 + 1\right]^{1/2}}{\dfrac{\eta}{\cos\delta} + 1} \tag{I-10}$$

Using the approximation in eq. (I-9), we obtain

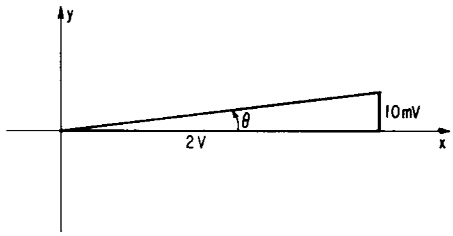

Fig. 4. Uncertainty in the horizontality condition.

$$\frac{C_{A\simeq B\cos\delta}}{A} \simeq \frac{\tan\delta\left[1+\left(\frac{\tan 2\theta}{\tan 2\delta}\right)^2\right]^{1/2}}{1+\frac{\cos 2\delta \tan 2\theta}{1+\cos 2\delta}} \tag{I-11}$$

We distinguish two separate regimes:

(a) *$\tan\theta \ll \tan\delta < 1$.* For this region, eq. (I-11) becomes

$$\frac{C_{A\simeq B\cos\delta}}{A} \simeq \tan\delta\{1-\theta\} \tag{I-12}$$

The error in our measurement of tan δ is thus given by θ, the error in establishing perfect "horizontality." For these larger values of tan δ ($0.005 < \tan\delta < 0.2$), we estimate our uncertainty to be certainly less than $\theta = 0.005$ (±10 mV on the y-axis when the x-axis amplitude is 2 V, see Fig. 4). This leads to insignificantly small errors.

(b) *$\tan\theta \simeq \tan\delta \ll 1$* ($0.001 < \tan\delta < 0.005$). For this region, we obtain

$$\frac{C_{A\simeq B\cos\delta}}{A} \simeq \tan\delta\left[1+\left(\frac{\theta}{\delta}\right)^2\right]^{1/2} \tag{I-13}$$

For these smaller values of δ, we estimate our uncertainty to be less than $\theta = 0.002$ (± 4 mV out of 2 V). (This smaller value for θ results from a combination of our observing ellipses which are less "fat" and hence more easily viewed and our switching to a more sensitive scale.) This may lead to a noticeable error in measuring tan δ as shown in eq. (I-13). However, this value of $\theta = 0.002$ is certainly a conservative overestimate as seen in the results shown in the paper. The few scattered points seen in the small tan δ region could reflect this error. A higher sensitivity could be achieved through additional filtering to reduce the noise (see Appendix II) and a more sensitive scale (our most sensitive scale at present is 50 mV/cm on the oscilloscope). However, the results we obtained were of sufficient accuracy that we did not implement these further improvements.

In conclusion, we have shown in this Appendix that the inability to attain perfect "horizontality" is not a serious problem and leads to negligible errors in our measurements of tan δ.

A comparable error analysis can be made of the original Rheovibron method, using eqs. (4) and (5) of the main text. If we define

$$\phi \equiv \frac{A-B}{B} \ll 1$$

to be the uncertainty in establishing the condition $A = B$ (ϕ is comparable to our θ or η), then we obtain

For *$\phi \ll \delta < 1$:*

$$\frac{C_{A\simeq B}}{A} \simeq 2\sin\frac{\delta}{2}\left(1-\frac{1}{2}\phi\right)$$

For *$\phi \simeq \delta \ll 1$:*

$$\frac{C_{A\simeq B}}{A} \simeq 2\sin\frac{\delta}{2}\left[1-\left(\frac{\phi}{\delta}\right)^2\right]^{1/2}$$

which are comparable to eqs. (I-12) and (I-13), respectively.

The increased sensitivity of our method arises from the smaller errors inherent to the "horizontality" approach of eq. (8) as compared to the "equal amplitude" approach of eq. (5). In other words, θ is smaller than ϕ, and this leads to more accurate values for the loss tangent.

Appendix II

The difference signal, eq. (3), is very small for small phase shifts, and hence noise may present significant problems. In this section, we show that use of a filter, even if it introduces a relatively large phase shift to the difference signal, nonetheless has a negligible effect on the desired loss tangent measurement.

The difference signal is given by [see eq. (3)]

$$\Delta(t) = C \cos (wt + \beta) \tag{II-1}$$

where

$$C = [(A - B \cos \delta)^2 + (B \sin \delta)^2]^{1/2} \tag{II-2}$$

and

$$\tan \beta = \frac{-B \sin \delta}{A - B \cos \delta} \tag{II-3}$$

Let us now suppose that this difference signal passes through a filter which introduces an additional phase shift ψ and attenuation ξ:

$$\Delta^f(t) = \xi C \cos (wt + \beta + \psi) \tag{II-4}$$

$$= \xi C \cos (\beta + \psi) \cos wt - \xi C \sin (\beta + \psi) \sin wt$$

We then place the filtered difference signal, $\Delta^f(t)$, instead of $\Delta(t)$ across the y-axis of the oscilloscope while keeping the strain signal, eq. (1), across the x axis. When we establish the "horizontality" condition by adjusting the gain of the stress signal, we make the coefficient of cos wt in eq. (II-4) vanish:

$$\xi C \cos (\beta + \psi) = 0 \tag{II-5}$$

Thus, using eqs. (II-5) and (II-3), we obtain

$$\tan \psi = \frac{1}{\tan \beta} = \frac{-(A - B \cos \delta)}{B \sin \delta} \tag{II-6}$$

When we calculate the phase angle by dividing the amplitude of the filtered difference signal by the amplitude of the strain signal, we obtain

$$\frac{C^f{}_{A=B\cos\delta}}{A} = \frac{\xi C \sin (\beta + \psi)}{A} \tag{II-7}$$

But from eq. (II-5), sin $(\beta + \psi) = 1$; and using eqs. (II-2) and (II-6), we obtain

$$\frac{C^f{}_{A=B\cos\delta}}{A} = \frac{\xi B \sin \delta}{A} [1 + \tan^2\psi]^{1/2}$$

Further use of eq. (II-6) yields

$$\frac{C^f{}_{A=B\cos\delta}}{A} = \frac{\xi \tan \delta [1 + \tan^2\psi]^{1/2}}{1 - \tan \delta \tan \psi} \tag{II-8}$$

We consider two cases:

(a) *$\tan \delta \ll \tan \psi \ll 1$.* Retaining lowest order terms, we obtain

$$\frac{C^f{}_{A=B\cos\delta}}{A} \simeq \xi \tan \delta \left\{1 + \frac{1}{2} \tan^2\psi\right\} \tag{II-9}$$

Since $\xi \simeq 1$ and tan ψ is very small, the error in measurement of tan δ due to filtering of the difference signal is negligibly small. In our experiments, we have used a filter for which tan $\psi = 0.034$. The resultant error due to the $\frac{1}{2}\tan^2 \psi$ term is thus seen to be less than 0.1%.

(b) *$\tan \psi \lesssim \tan \delta < 1$.* For this case, eq. (II-8) becomes

$$\frac{Cf_{A=B\cos\delta}}{A} \simeq \xi \tan \delta\{1 + \tan \delta \tan \psi\} \qquad \text{(II-10)}$$

The maximum value of tan δ is roughly 0.2. The resultant error due to the tan δ tan ψ term is thus seen to be less than 0.7%.

Appendix III

In this Appendix, we describe the modulus measurement. The amplified strain signal is given by

$$A \text{ (volts)} = \epsilon_0 f(\epsilon) D(\epsilon) L \qquad \text{(III-1)}$$

where A is the amplitude of the amplified strain gauge signal [eq. (1)], ϵ_0 is the strain amplitude, $f(\epsilon)$ is the amplification of the strain signal, $D(\epsilon)$ is the strain gauge conversion factor (volts/cm), and L is the sample length. The amplitude of the amplified stress gauge signal is given by

$$B \text{ (volts)} = \sigma_0 f(\sigma) D(\sigma) A_{cs} \qquad \text{(III-2)}$$

where B is given in eq. (2), σ_0 is the stress amplitude, $f(\sigma)$ and $D(\sigma)$ are the corresponding amplification and conversion factors for the stress signal, and A_{cs} is the sample cross section. Thus,

$$\frac{A\sigma_0}{B\epsilon_0} = \frac{f(\epsilon)D(\epsilon)L}{f(\sigma)D(\sigma)A_{cs}} \qquad \text{(III-3)}$$

In using the Rheovibron to measure the modulus of a sample, one adjusts B to equal A [eq. (4)] and thus obtains

$$E^* \equiv \frac{\sigma_0}{\xi_0} = \text{RHS of eq. (III-3) when } A = B. \qquad \text{(III-4)}$$

A calculation of the RHS (right-hand side) of eq. (III-3) when $A = B$ thus yields the complex modulus E^*. Furthermore, the magnitude of A or B is not needed in the determination of the complex modulus once the $A = B$ condition is satisfied. The only information required are geometric terms, amplification factors, and conversion factors.

Using our "horizontality" method, however, we set $A = B \cos \delta$ [eq. (6)] and obtain

$$E' \equiv \frac{\sigma_0}{\epsilon_0} \cos \delta = \text{RHS of eq. (III-3) when } A = B \cos \delta \qquad \text{(III-5)}$$

Thus, a calculation of the RHS of eq. (III-3) when $A = B \cos \delta$ yields the storage modulus E' directly, instead of the complex modulus E^*. The magnitude of A or B is again not required for a determination of E' as long as the "horizontality" condition ($A = B \cos \delta$) is satisfied.

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